$$u' = \frac{\Gamma}{2\sqrt{2\pi lL}}, \ \Gamma = \langle (\oint w dl)^2 \rangle^{1/2}, \ u' \equiv \langle w^2 \rangle^{1/2}.$$

Here w is the velocity fluctuation on a contour element $dl$. It is legitimate to propose that the measure of rotational excitation in a vortex is determined by parameter h, similar to the Planck constant in quantum physics — an analog of our h — serving as the measure of the inner angular momentum of a rotating particle. We then have $h \sim \rho\Gamma$ (in a nonvortical stream $h \to 0$ as $\Gamma \to 0$). On the basis of this estimate we obtain $\mu_T = \xi h$ and $\xi \sim 0.5(l/\pi L)^{1/2}$. For instance, $\xi = 0.1$ when $L = 5l$. The circulation Γ is expressed here in the "isotropic turbulence" approximation and, therefore, the preceding treatment is appropriate when no strong inhomogeneities occur in the fluctuations as, for example, in free turbulent streams.

## NOTATION

Here ψ is the wave function; $a$, wave amplitude; b, wave phase; U, modulus of the velocity; ρ, density (incompressible fluid); h, "quantum" parameter; x, y, longitudinal coordinate and the transverse coordinate in the mixing layer; and t, time.

## LITERATURE CITED


1. Ya. A. Vagramenko, "Wave theory of turbulence," Preprint, Institute of Problems in Mechanics, Academy of Sciences of the USSR, No. 137 (1980).
2. A. Messiah, Quantum Mechanics, Vol. 1, Halsted Press (1961).
3. A. A. Townsend, The Structure of Turbulent Shear Flow, Cambridge Univ. Press (1975).
4. E. Jahnke, F. Emde, and F. Lesch, Special Functions [Russian translation], Nauka, Moscow (1968).
5. R. P. Patel, "Experimental study of plane mixing zone," Raketn. Tekh. Kosmn., 2, No. 1, 79-84 (1973).
6. M. Van Dyke, Perturbation Methods in Fluid Mechanics, Parabolic Press (1975).
7. I. Wygnanski and H. E. Fiedler, "Two-dimensional mixing region," J. Fluid Mech., 41, Part 2, 327-361 (1970).


# FLOW MODEL OF A BOILING LIQUID IN NOZZLES


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UDC 532.529:536.423.1



A nonequilibrium mathematical model of flow is constructed for a boiling liquid in nozzles. The theoretical results are compared with empirical results for the case of flow of boiling water.


Present theoretical models of flow of a boiling liquid in nozzles (tubes) mainly aim to determine the maximum flow rate. A critical survey of these models is contained in [1, 2]. It is seen that homogeneous equilibrium and metastable models are the most widely used, along with a model allowing for discrete phase flow with relative slip and simple models considering the thermodynamic nonequilibrium of the process using empirical coefficients. It was shown in [3] that the use of such models is limited by the complexity of the flow structure of a boiling liquid and the nonequilibrium of exchange processes between the phases. In connection with this, it is important that a theoretical model of boiling liquid flow in nozzles be constructed which considers the structure of the flow and the effects of nonequilibrium of the interphase transfer. Such a model should describe the origination of the vapor phase in the liquid flow and the combined flow of the vapor and liquid phases.

Well-known experimental investigations of the structure of a boiling liquid flow in Laval nozzles [4-8] with moderate initial parameters show that boiling begins primarily on the nozzle walls. The method in [9] is used to determine the intensity of this vapor formation. It is assumed that vapor bubbles are generated until the vapor content of the mixture reaches a value which is limiting for the existence of a bubble structure ($\alpha \simeq 0.74$).


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The joint flow of vapor and liquid phases is described using the nonequilibrium model of a multiphase medium in [10]: a dual-velocity dual-temperature model of two-phase flow in a unidimensional, steady-state approximation. We will obtain the mathematical flow model in formulating the direct problem (calculation of the flow parameters in a channel of a specified geometry). We will assume that inversion of the bubble structure into a drop structure takes place when a volume vapor content of 0.74 is reached in the mixture.

Suppose that the vapor–liquid flow is a moving, monodisperse mixture of liquid and vapor with spherical bubbles or drops uniformly distributed in the carrier phase. The vapor in the bubbles will be considered saturated. We will also assume that the flow is nonequilibrium with respect to temperature only in the bubble region and with respect to both temperature and velocity in the drop region.

We write the differential equations describing the motion of a vapor–liquid mixture with a drop structure:

$$\frac{dm_v}{dz}=\frac{d}{dz}(\rho_v^0 w_v f_v), \tag{1}$$

$$\frac{dm_l}{dz}=\frac{d}{dz}(\rho_l^0 w_l f_l), \tag{2}$$

$$(1-\alpha)\rho_l^0 w_l\frac{dw_l}{dz}=-(1-\alpha)\frac{dp}{dz}+X_f-X_{\tau l}, \tag{3}$$

$$\alpha\rho_v^0 w_v\frac{dw_v}{dz}=-\alpha\frac{dp}{dz}-X_f-X_{\tau v}+\frac{1}{f}\frac{dm_v}{dz}(w_l-w_v), \tag{4}$$

$$(1-\alpha)\rho_l^0 w_l\frac{di_l}{dz}=(1-\alpha)w_l\frac{dp}{dz}-q_{l\sigma}, \tag{5}$$

$$\alpha\rho_v^0 w_v\frac{di_v}{dz}=\alpha w_v\frac{dp}{dz}+\frac{1}{f}\frac{dm_v}{dz}\frac{(w_l-w_v)^2}{2}-q_{v\sigma}+X_f(w_v-w_l), \tag{6}$$

$$q_{v\sigma}+q_{l\sigma}=\frac{1}{f}\frac{dm_v}{dz}l(p). \tag{7}$$

The last equation describes the balance of the inflow and outflow at the phase boundary corresponding to a quasiequilibrium scheme of phase transition [11].

We will assign laws of mechanical phase interaction and phase friction against the channel wall similar to those in [12]. System (1)-(7) has a different form in describing the flow of a vapor–liquid mixture with a bubble structure:

a) a single equation of motion is written for the mixture;

b) the terms connected with velocity nonequilibrium are absent from the equation describing heat flow to the vapor phase;

c) the system is supplemented by an equation describing the change in the number of bubbles per unit volume of the mixture due to their separation from the channel walls

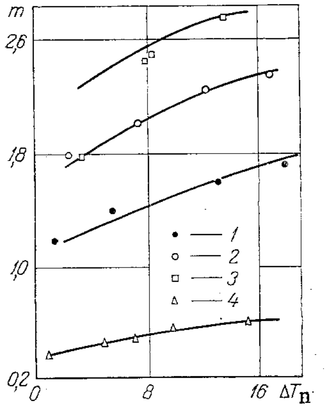
$$\frac{J\Pi}{fw_l}=\frac{dN}{dz},$$

with averaging of the bubble diameter over the volume being done at each step in the integration of system (1)-(7)

$$d=\sqrt[3]{\frac{6\alpha}{\pi N}}.$$

After solving (1)-(7) relative to the derivatives, we represent the pressure gradient in the form

$$\frac{dp}{dz}\sim\frac{\varphi+\frac{1}{f}\frac{df}{dz}}{1-\frac{w_v^2}{a_f^2}}, \tag{8}$$



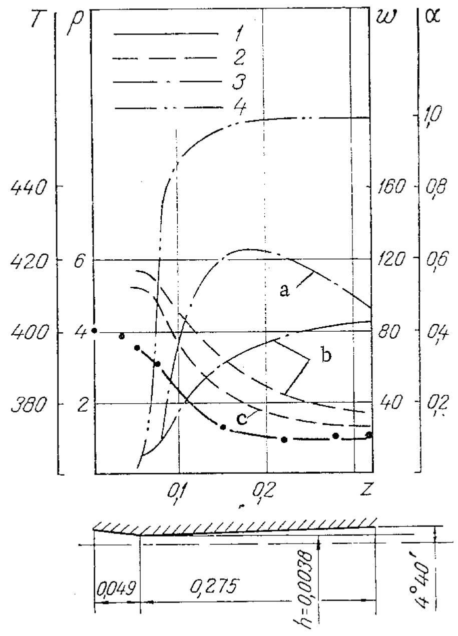


Fig. 1.    Fig. 2.

Fig. 1. Comparison of experimental [1) water flow rate at $p^* = 5.9$; 2) 11.8; 3) 17.6 [7]; 4) 4 [8] bars] and theoretical flow-rate characteristics of a Laval nozzle. m, kg/sec; $\Delta T_n$, °K.

Fig. 2. Theoretical distributions of parameters along the Laval nozzle [1) p; 2) T; 3) w; 4) α]: a) vapor; b) liquid; c) saturation temperature; points — experimental distribution of static pressure along the nozzle [8], $p^* = 4$ bars; $\Delta T_n = 0.5$°K. w, m/sec; z, m.

where $a_f$ is the frozen speed of sound in the mixture; φ is a function defining the interphase interaction and the friction of the mixture against the nozzle wall.

Within the framework of the chosen model, we have

$$a_f^2 = \begin{cases} \dfrac{\rho_v^0}{\alpha\,[\alpha\rho_v^0 + (1-\alpha)\,\rho_l^0]}\,\dfrac{dp}{d\rho_{vs}} & \text{at} \quad \alpha < 0{,}74, \\ \left(1 + \dfrac{1-\alpha}{\alpha}\,\dfrac{\rho_v^0}{\rho_l^0}\,\dfrac{w_v^2}{w_l^2}\right)\left(\dfrac{\partial p}{\partial \rho_v^0}\right)_{S_v} & \text{at} \quad \alpha \geqslant 0{,}74 \end{cases} \tag{9}$$

(the liquid is incompressible, $\rho_l^0 = \rho_l^0(T_l)$). Equations (8)-(9), including the notation, agree with those obtained earlier in [3].

Let us stop here to specify the laws of interphase transfer. We will determine the heat flux $q_{l\sigma}$ from the liquid to the phase boundary using Scriven's well-known solution [13], obtained with the assumption that bubble growth rate is limited by the flow of heat from the liquid to the phase boundary. The analytical approximation in [14] yields a relation of the type in [10]:

$$\mathrm{Nu}_{l\sigma} = 3.9\mathrm{Ja}\left[1 + \frac{1}{2}\left(\frac{\pi}{6\,\mathrm{Ja}}\right)^{2/3} + \frac{\pi}{6\,\mathrm{Ja}}\right].$$

Using an equation such as (5) for the flow of heat to the discrete phase, the assumption of saturation of the vapor in the bubbles allows us to obtain the relationship between heat flux $q_{v\sigma}$ and pressure gradient dp/dz.

In the case where the two-phase mixture has a drop structure, heat flow from the vapor to the surface of a drop with the temperature $T_s(p)$ is usually described by the relation for heat exchange involving a single spherical particle with an infinite flow of the carrier

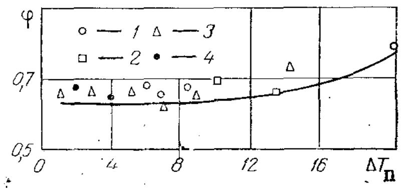



Fig. 3.

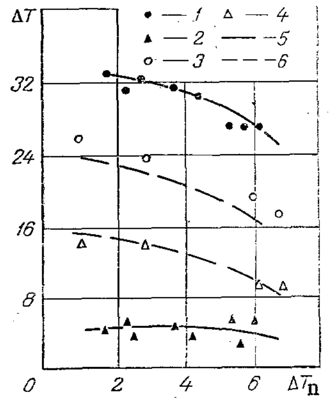



Fig. 4.

Fig. 3. Comparison of experimental [8] and theoretical values of the velocity coefficient of a Laval nozzle: 1) p* = 3; 2) 3.65; 3) 4; 4) 5 bars.

Fig. 4. Comparison of experimental [8] and theoretical values of superheating of the liquid phase at the nozzle outlet ($\Delta T_s$) and difference in liquid-phase temperature at the nozzle inlet and outlet ($\Delta T_{res}$): 1) $\Delta T_{res}$, L/d = 48; 2) $\Delta T_s$, L/d = 48; 3) $\Delta T_{res}$, L/d = 18; 4) $\Delta T_s$, L/d = 18; 5) calculation, L/d = 48; 6) calculation, L/d = 18. ΔT, °K.

phase [6, 10]:

$$\mathrm{Nu}_{v\sigma} = 2 + 0.6\mathrm{Pr}_v^{0.33}\mathrm{Re}^{0.5}.$$

To determine the heat flux from the superheated drop to the phase boundary $q_{l\sigma}$, it is necessary to know the temperature gradient on the drop surface. Thus, at each step of integrating system (1)-(7), we numerically solved the equation of heat conduction in the spherical drop with first-order boundary conditions, each time finding the temperature profile in the drop and the temperature gradient on its surface. We used an implicit difference scheme and found the solution by the trial-run method.

The problem as a whole was solved by means of the following "adjustment" algorithm: with fixed stagnation parameters at the nozzle inlet, we assigned the pressure in the minimum cross section, thereby determining the rate of flow of the liquid and its superheating (assuming that there was no vapor formation in the convergent part of the nozzle). We then used the Runge–Kutta method, with automatic selection of the interval, to numerically solve the system of equations describing the flow of the vapor–liquid mixture. We used the method of successive approximations to find the value of flow rate at which, with a specified accuracy, the following boundary condition is satisfied

$$p = p_\infty \quad \text{at} \quad z = L$$

or at which one of the conditions of realization of the critical flow regime is satisfied [3]:

$$\frac{dp}{dz} = \frac{0}{0} \quad \text{at} \quad z < L, \quad \frac{dp}{dz} = -\infty \quad \text{at} \quad z = L.$$

In the range of outflow parameters investigated, a typical condition of realization of the critical regime is the occurrence during the solution of a singular (Sedlov) point in the bubble-structure flow region. This is connected with the fact that, in the chosen model ($w_v = w_l$ at $\alpha < 0.74$, inertia of the liquid in the radial direction neglected), the frozen speed of sound in the mixture (9) is of the same order of magnitude as the equilibrium velocity.

Experimental data from [7, 8] is used for comparison with the estimates. Figure 1 compares empirical and theoretical flow-rate characteristics of a Laval nozzle operated on boiling water. The divergence here is no greater than 4-5% (except for the point corresponding to $p^* = 17.6$ bars, $\Delta T_n = 3.1°K$). Figure 2 shows typical theoretical distributions of the flow parameters along the divergent part of the nozzle. Also shown are experimental values of static pressure measured at different sections of the nozzle [8]. Analysis of the theoretical distributions of the flow parameters permits the following conclusions:

a) the flow has a vapor-drop structure for the main part of the nozzle ($\alpha \geqslant 0.74$);

b) the flow is characterized by substantial interphase slip ($w_v - w_l$) and superheating of the liquid.

Figure 3 compares theoretical and experimental values of the nozzle velocity coefficient $\varphi$ [5, 6] in subcritical (within the framework of the chosen model) flow regimes. The divergence does not exceed 8-10%. Given fixed $p^*$, a reduction in the initial temperature of the liquid is accompanied by an increase in the coefficient $\varphi$, which is due to an increase in the fraction of the enthalpy drop recorded in the pure-liquid flow region (a similar result was described in [6]).

The work [8] presented measurements of the temperature of heated water at the outlet of Laval nozzles of different relative lengths L/d.

Thus, within the parameter range investigated, the characteristics of Laval nozzles working on boiling water can be reliably predicted.

## NOTATION

p, pressure, bars; T, temperature, °K; $\rho$, density, kg/m$^3$; w, velocity, m/sec; m, mass flow rate, kg/sec; $l$, heat of transformation, J/kg; f, cross-sectional area of channel, m$^2$; $\alpha$, volume vapor content; i, enthalpy, J/kg; S, entropy, J/kg·°K; $X_f$, drag of drops per unit volume of the mixture, N/m$^3$; z, axial coordinate of nozzle, m; N, number of bubbles (drops) per unit volume of mixture, m$^{-3}$; $\Pi$, channel perimeter, m; J, capacity of surface centers of vapor formation, sec$^{-1}$·m$^{-2}$; $\Delta T_n$, initial subheating, K; L, nozzle length, m; d, diameter of bubbles, characteristic dimension of minimum nozzle cross section, m. Indices: $l$, liquid; v, vapor; $\sigma$, phase boundary; s, saturation parameters; $\infty$, parameters at infinity; *, stagnation parameters.

## LITERATURE CITED

1. V. V. Fisenko, Critical Two-Phase Flows [in Russian], Atomizdat, Moscow (1978).
2. V. D. Keller, "Critical two-phase flows in hydraulic channels," Vopr. Raketnoi Tekh., No. 12, 35-51 (1971).
3. K. I. Soplenkov, "Study of steady-state and transient critical outflow of gas- and vapor—liquid mixtures," Author's Abstract of Candidate's Dissertation, Physicomathematical Sciences, Moscow (1979).
4. G. A. Mukhachev, B. N. Pavlov, and V. G. Tonkonog, "Flow of a vaporizing liquid in nozzles," Tr. Kazan. Aviats. Inst., No. 158, 50-54 (1973).
5. V. A. Zysin, G. A. Barinov, V. A. Barilovich, and N. N. Parfenova, Boiling Adiabatic Flows [in Russian], Atomizdat, Moscow (1976).
6. G. V. Tsiklauri, V. S. Danilin, and L. I. Seleznev, Adiabatic Two-Phase Flows [in Russian], Atomizdat, Moscow (1973).
7. M. A. Koronkevich, "Flow-rate characteristics of Laval nozzles with a flow of boiling water," Preprint 15-77, Novosibirsk (1977).
8. V. N. Blinkov, I. I. Petukhov, M. A. Bespyatov, et al., "Experimental study of the flow of boiling water in a Laval nozzle," in: Gas Thermodynamics of Multiphase Flows in Power Plants [in Russian], Vol. 4, Kharkov Aviation Institute, Kharkov (1981), pp. 71-78.
9. A. A. Avdeev, V. N. Maidanik, and V. K. Shanin, "Method of calculating boiling adiabatic flows," Teploenergetika, No. 8, 67-69 (1977).
10. R. I. Nigmatulin, Principles of the Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
11. D. A. Labuntsov and T. M. Muratova, "Physical and methodological principles of formulating heat- and mass-transfer problems for phase transformations," in: Heat and Mass Transfer [in Russian], Vol. 2, Part 1, Minsk (1972), pp. 112-122.
12. A. I. Borisenko, V. G. Selivanov, and S. D. Frolov, "Design and experimental study of a gas—liquid nozzle with a substantial content of liquid in the gas," in: Problems of the Gas Thermodynamics of Power Plants [in Russian], Vol. 1, Kharkov Aviation Institute, Kharkov (1974), pp. 83-93.

13. L. E. Scriven, "On the dynamics of phase growth," Chem. Eng. Sci., 1, 1-13 (1959).
14. D. A. Labuntsov, B. A. Kol'chugin, V. S. Golovin, et al., "High-speed-film study of bubble growth during the boiling of saturated water in a broad pressure range," Teplofiz. Vys. Temp., 2, No. 3, 446-453 (1964).

# EFFECT OF BOUNDARY-LAYER INJECTION ON THE DRAG OF AN AXISYMMETRIC BODY IN A HYPERSONIC IMPERFECT-GAS FLOW

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The article examines a similarity problem of hypersonic flow of an imperfect gas about an axisymmetric body in the presence of boundary-layer injection.

We will study the basic principles underlying the effect of gas injection over a solid surface into a boundary layer on the drag of the axisymmetric body in a hypersonic gas flow by examining flow about a body with its generatrix described by the power relation $r_w(x) = ax^{3/4}$. It was shown in [1] that, in this case, the problem of the viscous interaction of the nonpermeable surface of the body with the imperfect heat-conveying gas has the property of similitude, and the gasdynamic parameters in the boundary layer depend on a single variable

$$\eta = \frac{\sqrt{\mathrm{Re}} \int_{r_w}^{r} \rho r dr}{2M_\infty \sqrt{\xi}}$$

These parameters are determined from a system of ordinary differential equations. When gas is injected over the surface of the body into the boundary layer, the similarity property of the problem is preserved if the distribution of the mass injection rate along the generatrix of the body is determined by the relation

$$(\rho v r)_w = a^4 \frac{\chi}{(M_\infty a)^2} \sqrt{1 + kJ(\infty)} \frac{3\sqrt{c}}{4} \alpha. \tag{1}$$

In accordance with [1], the following notation is adopted:

$$\chi = \frac{M_\infty^3}{\sqrt{\mathrm{Re}}}; \quad \xi = \frac{1}{2} \int_0^x p_\delta r_w^2 dx; \quad \eta = \frac{\sqrt{\mathrm{Re}}\, u_\delta}{2M_\infty \sqrt{\xi}} \int_{r_w}^{r} \rho r dr; \quad p_\delta = c \left(\frac{dr^*}{dx}\right)^2.$$

If a gas different from the gas of the hypersonic flow is injected into the boundary layer, then, besides (1), we need to assume that the viscosity coefficient of these gases under these conditions is a power function of temperature and that the exponent is the same for each gas. Then the differential equations to which the problem is reduced take the form

$$\frac{\partial}{\partial\eta}\left(YN \frac{\partial u}{\partial\eta}\right) + f \frac{\partial u}{\partial\eta} = 2m(g - u^2),$$

$$\frac{\partial}{\partial\eta}\left(\frac{YN}{\mathrm{Pr}} \frac{\partial g}{\partial\eta}\right) + f \frac{\partial g}{\partial\eta} = -\frac{\partial}{\partial\eta}\left(YN\left(1 - \frac{1}{\mathrm{Pr}}\right)\frac{\partial u^2}{\partial\eta}\right) - \frac{\partial}{\partial\eta}\left(\frac{1}{\mathrm{Sm}}\left(1 - \frac{1}{\mathrm{Le}}\right)\sum_i \frac{h_i - h_i^0}{H_\delta} \frac{\partial c_i}{\partial\eta}\right), \tag{2}$$

$$\frac{\partial}{\partial\eta}\left(\frac{YN}{\mathrm{Sm}} \frac{\partial c_i}{\partial\eta}\right) + f \frac{\partial c_i}{\partial\eta} = 0$$

The following notation is introduced here:

$$u = \tilde{u}/u_\delta; \; f = -\alpha + \int_0^\eta u d\eta, \; g = H/H_\delta, \; N = \rho\mu/pM_\infty^2, \; Y = r^2/r_w^2 = 1 + kJ(\eta), \; J(\eta) = \int_0^\eta F(h)\, d\eta, \; m = \left(\frac{1}{2} - n\right)\frac{\gamma_\delta - 1}{2\gamma_\delta},$$

$$r^* = r_w(1 + kj(\infty)), \quad n = 3/4, \qquad k = 2M_\infty \sqrt{\xi} / \sqrt{\mathrm{Re}}\, pr_w^2$$

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